

DREXEL MATHEMATICS PHD PROGRAM HANDBOOK

1. PROGRAM OVERVIEW

Obtaining a PhD in Mathematics at Drexel University has the following major components:

- (1) Coursework.
- (2) The qualifying (or preliminary) exam.
- (3) Selection of doctoral advisor (a.k.a. dissertation or thesis advisor, or supervising professor).
- (4) The candidacy exam.
- (5) Research.
- (6) The dissertation (also called the thesis).
- (7) The dissertation defense.

-*Coursework*: To obtain a PhD, students must complete a minimum of 90 degree credits, of which at least 45 must be obtained by coursework (the remaining credits are obtained *via* “research credits”). Drexel Mathematics graduate courses are 3 credits each, so that is to say 15 classes. Of these, 18 credits (6 classes) must be MATH 504, 505, 506, 533, 630 and 633. The remaining 27 credits (9 classes) may be any graduate level mathematics course, with the exception of MATH 544, 545 or 546. Some of these credits may be taken as classes outside of the mathematics department, but students must receive prior departmental approval (from the Department Graduate Advisor) for these to count towards the degree. A minimum GPA of 3.0 must be maintained. Failure to do so results in the Graduate College placing the student on *academic probation*. See the following site for details:

<https://drexel.edu/provost/policies/graduate-student-academic-standing-policy/>

Most coursework is completed in the first two years of the program. Students are required to take minimally one course per academic year while full-time. Note that students who complete the 45 credits worth of coursework are eligible to receive a Master’s (MS) in Mathematics. Graduate College policy states that “all doctoral students who do not already have a master’s degree in their area of study may, with the approval of their program department, apply for such a degree when they complete the requirements for it. The application must be made in the quarter in which the student is eligible. The degree cannot be awarded retroactively.”

-*The qualifying exam*: This is a four hour long written exam and the material covers the topics of *real analysis* and *linear algebra*. The subject matter parallels the content of MATH 504, 505 (analysis) and 506 (linear algebra). The specific list of topics on the exam can be found at the end of this handbook. Students are afforded two opportunities to pass this exam. The first takes place immediately following the Spring Quarter at the end of their first year. Should students not pass on the first attempt, a second attempt takes place

immediately prior to the start of the Fall Quarter of their second year. Students who take but do not pass the second attempt do not continue in the PhD program; if they are otherwise in good academic standing they are permitted to continue in the MS program. Scheduling and administering the exam is done by the Department Graduate Advisor. Copies of old exams are available from the Department Graduate Advisor upon request.

-Selection of doctoral advisor: Students who pass the qualifying exam then begin the process of selecting their doctoral advisor. The doctoral advisor must be a tenured or tenure-track faculty member of the Drexel Mathematics Department. It is possible to have multiple “co-advisors.”

Picking a doctoral advisor is an extremely important aspect of graduate school, as the doctoral advisor plays a major educational and scientific role. They guide the student’s research, mentor them on professional matters and lead the evaluation and assessment of the student’s scholarly work, most notably their dissertation. Student’s should balance their scholarly interests with interpersonal dynamics when considering the selection.

Here is the rough process for selecting an advisor. Students familiarize themselves with the tenure/tenure-track faculty and their areas of research. This can be done by attending seminars, taking courses, examining the faculty members’ websites or just knocking on doors and asking questions. Students can also ask the Department Graduate Advisor for guidance on this. Once the student has an idea of who they would like to work with, the student then asks the faculty member if they are willing to be their advisor. Faculty are not obligated to say yes. If they do not, another faculty member will need to be asked. Final determination of the doctoral advisor should be made no later than the end of the Fall Quarter of the student’s second year.

Circumstances may arise in which a student needs to switch advisors. Such a change can have substantive outcomes on the student’s research and dissertation and should not be undertaken lightly. To effect such a change, the student notifies the Departmental Graduate Advisor who then oversees the change.

-The candidacy exam: Upon selection of the doctoral advisor, the student begins preparing for the candidacy exam. This examination takes the form of an oral presentation followed by a question/answer period, together with a written report corresponding to the presentation. The exam must take place prior to the start of the student’s third year. The exam is meant to prepare the student for mathematics research and to assess their readiness for the same. The subject matter of the exam is determined by the doctoral advisor in conjunction with the student. The exam is administered by the student’s *dissertation advisory committee* (DAC), whose composition is described the Drexel’s Graduate College as follows:

The [DAC] must consist of at least five members, at least three of whom must be currently tenured or tenure-track Drexel faculty members. At least two of the committee members must be from outside of student’s primary specialization area. At least one of the committee members must be from outside of student’s department, preferably from outside the university. Full-time, non-tenure track Research Faculties are also eligible to serve on the Dissertation Advisory Committee, including as Co-Supervising Professor.

Note that, though it is not explicitly specified, the doctoral advisor is one of the five committee members and serves as chair of the examination. The student should confer with the doctoral advisor about choosing the members, especially the member who hails from outside the Mathematics Department. The written report should be distributed to the DAC a week prior to the date of the exam.

The student either passes the candidacy exam (who then becomes a *doctoral candidate*) or not. If the latter, the DAC is to provide a course of action for the student the DAC has a large degree of freedom in this regard. They can recommend that the student make another attempt at the candidacy exam, recommend that the student not continue in the PhD program or anything in between.

There is a fair bit of paperwork associated with this exam required by the Graduate College. These are

- the Plan of Study and Supervising Professor Appointment Form (Form D-1, to be completed by the student prior to the candidacy exam),
- the Doctoral Candidacy Examination Report (Form D-2, to be completed by the supervising professor after the candidacy exam) and
- the Doctoral Candidacy Examination Member Report (Form D-2A, to be completed DAC members after the candidacy exam).

These forms can be found at the Graduate College website and are currently located at:

<https://drexel.edu/graduatecollege/forms-policies/forms/>

Each of these forms is to be returned to the Graduate College and contains instructions on who is to do so.

-*Research*: Mathematical research can take many forms. Writing rigorous proofs of new theorems, devising mathematical models for physical phenomena and developing and implementing new computational algorithms are just a few possibilities. Given this variety, a great deal of latitude is given the doctoral advisors in shaping, guiding and assessing a PhD student's research program.

The period of research generally begins during preparation for the candidacy exam and goes through the writing of the dissertation and its defense. After year two, most of the student's academic time will be spent on research: as stated above, after completing the 45 credits of coursework, only one course per academic year needs to be taken. At this point, students are assigned "research credits," up to 9 per quarter. Students and their doctoral advisors should meet regularly and set concrete goals for making progress.

-*The dissertation*: Drexel's Graduate College states that a "dissertation is required for doctoral degree candidates and must be based on original research and clearly demonstrate the candidate's ability to work at the frontiers of the field." The research carried out by the student forms the backbone of the dissertation, though it may also contain sections of background material. The doctoral advisor and the DAC play an important role in directing the students efforts in the composition of the dissertation. Some dissertations are long, some are short, but all must contain novel scholarly work. The Graduate College has policies on the format and style of the dissertation. These can found at:

<https://drexel.edu/graduatecollege/academics/thesis-and-dissertation/>

The dissertation is to be read and evaluated by the doctoral advisor and the DAC. Corrections of and changes to the dissertation requested by DAC are to be implemented by the student. The final version is then approved by the DAC. Paperwork to this end is to be included in dissertation and can be found at the above link.

The dissertation defense: The defense is a public presentation of the work contained in the dissertation followed by a closed meeting with the student's DAC, who then ask questions related to the presentation of the dissertation as well as the dissertation itself. Note that the dissertation need not be fully complete prior to the defense, though ideally it is all but. A draft of the dissertation must be provided to the DAC two weeks prior to the defense. The defense provides a forum for final comments and changes to be requested by the DAC. The DAC must fill out and submit the Report of PhD Final Oral Defense (Form D-5) following the defense. This form can be found at:

<https://drexel.edu/graduatecollege/forms-policies/forms/>

A dissertation defense has a *chair*. The chair is to be selected from the DAC, though it need not be the doctoral advisor. The chair's role is primarily ceremonial: they introduce the student prior to the defense and oversee the question and answer section. They also manage the submission of the D-5, mentioned above.

2. OTHER POLICIES AND RESOURCES

-Duration of program The PhD program typically takes five years from start to finish. With approval of their doctoral advisor and the DAC, a student may complete their thesis and defense in fewer than five years. Students who wish to extend their time into a sixth year, or beyond, need to obtain the approval of their doctoral advisor and the Department Graduate Advisor. Beyond seven years, approval is needed from the Graduate College. Their policy on this can be found here

<https://drexel.edu/graduatecollege/forms-policies/policies-procedures-guidelines/time-to-completion/>

-Seminar policy: All full-time Mathematics PhD students who successfully pass the qualifying exam are to attend 5 departmental seminars/colloquia per quarter (Fall, Winter, Spring only). By attending such events, students become more involved in the broader mathematical community and begin to see the bigger picture of the current state of mathematical research, hopefully fostering both new research ideas and creating networking opportunities. The Department Graduate Advisor will have a list of the currently running seminars/colloquia, available on request, and they will also maintain a record of student attendance.

-Annual evaluation: In each year after they pass the qualifying exam, a PhD student's doctoral advisor, in concert with the Departmental Graduate Advisor, completes the "PhD Student Annual Evaluation Form." This form tracks the student's progress in coursework and research. For students who serve as teaching assistants, the evaluation also addresses their performance in the classroom and the Math Resource Center. If a student's academic progress is unsatisfactory, or their performance as a TA is, the evaluation will make note and provide feedback for future improvement. The evaluation is to be completed and delivered to the student prior to the start of the Fall Quarter.

-*Travel funds*: Students may wish to attend a conference or workshop to further their research goals. Funding for such activities may be requested from the Mathematics Department by asking the Department Graduate Advisor. At their discretion, some funding may be granted. For such events abroad, students can apply for an International Presentation Award from Drexel's Office of Global Engagement. The details can be found here:

<https://drexel.edu/global/research-funding/international-presentation-awards/>

-*The Department Graduate Advisor and the Graduate Program Manager*: The Mathematics Department has two people who oversee the graduate program. The first is the Department Graduate Advisor. This is a faculty member who handles most of the academic side of the program. Many of their responsibilities are described above. They are the main point of contact for students who have questions about academic and mathematical issues. The other person is the Graduate Program Manager. This is a non-faculty administrator who oversees the bureaucratic aspects of the program. They can help students register for classes, withdraw from classes, interface with the registrar or bursar's offices, find and fill out paperwork, access academic records and many more things besides. They are an indispensable resource for students.

-*The Graduate College*: Drexel's Graduate College has a number of policies which students should familiarize themselves with. Included are policies on course withdrawal, incomplete grades, academic standing and many other things. They can be found here:

<https://drexel.edu/graduatecollege/forms-policies/policies-procedures-guidelines/>

-*The Provost's Office*: The Provost's Office also many policies which affect graduate students. An overview can be found at: <https://drexel.edu/provost/policies/overview/>

-*International Student and Scholars Services*: This is an essential office for our international graduate students. Their website is:

https://drexel.edu/studentlife/get_involved/international_students_scholars/

3. YEAR BY YEAR TIMELINE

The following is a typical outline of how things proceed year to year. It is not an exact schedule and individual experiences may vary.

- Year 1:
 - Fall Quarter: Math 504, 505 + one elective.
 - Winter Quarter: Math 506, 630 + one elective.
 - Spring Quarter: Math 533, 633 + one elective.
 - Summer Quarter: the qualifying exam.
- Year 2:
 - Fall Quarter: three electives + select doctoral advisor.
 - Winter Quarter: three electives.
 - Spring Quarter: three electives.
 - Summer Quarter: the candidacy exam.
- Year 3: research, one elective taken.
- Year 4: research, one elective taken.
- Year 5:
 - Fall + Winter Quarters: research, one elective taken.

– Spring Quarter: dissertation completed, defense.

4. QUALIFYING EXAM TOPICS

Linear Algebra

- Gaussian elimination, echelon form/row reduced form, LU-decomposition, block matrices and their multiplication, permutation, elementary matrices, rank, linear (in)dependence, determinant, co-factors, Cramer's rule, Cayley Hamilton theorem.
- Vector spaces, subspaces, linear transformations, matrix representation, change of basis, injectivity, surjectivity, inverse.
- Orthogonality, projections, least square approximations and their normal equations, orthogonal bases, Gram-Schmidt orthogonalization, QR factorization, unitary matrices, Schur triangularization.
- Eigenvalues and eigenvectors, diagonalization, Jordan canonical form, spectral theorem for symmetric/normal matrices, Courant-Fischer theorem.
- Norms, condition numbers, Gersgorin disks.
- Positive semidefinite matrices, singular value decomposition, polar decomposition, Perron-Frobenius Theorem.
- Suggested textbooks: Roger Horn and Charles Johnson, *Matrix Analysis*. B. Noble and J. Daniel, *Applied Linear Algebra*. Kenneth Hoffman and Raymond Kunze, *Linear Algebra*. Sheldon Axler, *Linear Algebra Done Right*. Stephen Friedberg, *Linear Algebra*. Gene Golub and Charle Van Loan, *Matrix Computations*. L. Trefethen and D. Bau, *Numerical Linear Algebra*. Peter Lax, *Linear Algebra*. Peter Lancaster, Miron Tismenetsky, *The Theory of Matrices*.

Real Analysis

- Basic properties of real numbers developed from ordered field and completeness properties.
- Elements of set theory: cardinality, countable and uncountable sets.
- Elements of general metric spaces: convergence, completeness, compactness, connectedness, and their basic properties.
- numerical sequences and their convergence; lim sup and lim inf of a sequence.
- numerical series and their convergence: tests for convergence, unconditional and absolute convergence.
- continuous functions and their properties; uniform continuity.
- derivative and its properties; mean value theorem, Taylor expansions.
- Riemann (Stieltjes) integral and its properties.
- sequences and series of functions; pointwise and uniform convergence; Stone- Weierstrass Theorem; power series
- Calculus of functions of several variables; differentiation, integration and their basic properties: chain rule, inverse and implicit function theorems, change of variables in multiple integrals.
- Lebesgue integral: definition and its properties; fundamental convergence theorems.
- Suggested textbooks: Walter Rudin, *Principles of Mathematical Analysis*. Tom Apostol, *Mathematical Analysis*. R.P. Boas, *A Primer of Real Functions*. R.C.

Buck, *Advanced Calculus*. H.L. Royden, *Real Analysis*. Michael Spivak, *Calculus on Manifolds*.